because its axial velocity is very small. The vapor, being surrounded by a liquid film, is isothermal at its saturation temperature. At high qualities, when the continuous liquid film finally breaks, the remaining liquid is concentrated at the stagnation points of the secondary flow (3 and 9 o'clock) and the heat-transfer coefficient at other points decreases to the pure vapor coefficient. If this model is valid, we have the somewhat unusual situation of a phenomenon (the radial acceleration, in this case) effective only for that portion of the process where it is helpful and ineffective where it would be damaging.

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ON THE RADIATION SLIP BETWEEN ABSORBING–EMITTING REGIONS WITH HEAT SOURCES

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(*Received* 10 *January* 1966 *and in revised form* 20 *September* 1966)

SOME practical systems in which regions with sharply differing properties and energy sources are exchanging radiation are being extensively studied. Examples are "seeded" layers with flow and the various gaseous-core nuclear propulsion systems. Examination of these situations show that a discontinuity in temperature may exist in certain cases at the boundary between the regions. This is analogous in some ways to the often-noted temperature jump at the boundary of a gas abutted by a solid surface [1–3]. Such slips occur only where radiation is considered independent of conduction or where very low densities are present in the gas.

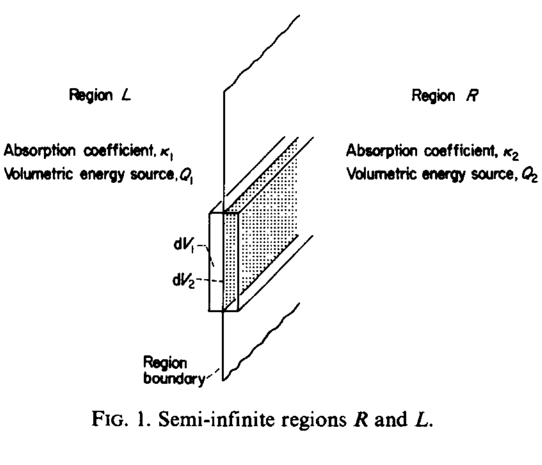

FIG. 1. Semi-infinite regions R and L.

The magnitude of the emissive power slip at a gas–gas interface will now be derived for a simple system.

Consider two semi-infinite regions as shown in Fig. 1. Each region has a given absorption coefficient κ and volumetric heat source strength Q. For simplicity, let Q be a different constant value throughout each region. Then, for an infinitesimal volume element dV_1 wholly within region L, the energy emitted must be equal to the radiation absorbed by the element plus the energy generated within it, or

$$\begin{aligned} 4\int_{\lambda=0}^{\infty} \kappa_{\lambda,1}\, e_{\lambda,1}\, dV_1\, d\lambda &= \int_{\lambda=0}^{\infty} \kappa_{\lambda,1} \Big[\int_{V_L} 4\kappa_{\lambda,e}\, e_{\lambda,e}\, F_{e-dV_1} \exp\Big(-\int_0^r \kappa_{\lambda,r} dr\Big) dV_e \\ &\quad + \int_{V_R} 4\kappa_{\lambda,e}\, e_{\lambda,e}\, F_{e-dV_1} \exp\Big(-\int_0^r \kappa_{\lambda,r}\, dr\Big) dV_e\Big] d\lambda dr \\ &\quad + Q_1\, dV_1 \end{aligned} \tag{1}$$

where e_λ is the local black spectral emissive power of the medium, F_{e-dV_1} is a shape factor between the emitting element dV_e and the absorbing element dV_1, and r is the radial position in spherical coordinates. The part of the equation within brackets represents the radiation incident on dV_1 in the wavelength interval $d\lambda$. Multiplying by the absorption coefficient $\kappa_{\lambda,1}$ and the mean path length through dV_1, denoted by dr, and then integrating over λ gives the total energy absorbed by dV_1. If $\kappa_{\lambda,1}$ is taken to be independent of wavelength and is a constant within the region, then equation (1) becomes

$$e_1\, dV_1 = A + \frac{Q_1\, dV_1}{4\kappa_1} \tag{2}$$

where

$$A = [\int_{V_L} \kappa_1 e_e F_{e-\mathrm{d}V_1} \exp(-\kappa_1 r)\,\mathrm{d}V_e + \int_{V_R} \kappa_2\, e_e\, F_{e-\mathrm{d}V_1} \exp(-\kappa_2 r)\,\mathrm{d}V_e]\,\mathrm{d}r$$

and A is proportional to the total energy incident on $\mathrm{d}V_1$.

A similar equation for a volume element $\mathrm{d}V_2$ at the boundary but wholly within region 2 is

$$e_2\,\mathrm{d}V_2 = A' + \frac{Q_2\,\mathrm{d}V_2}{4\kappa_2}. \tag{3}$$

The integral term in equation (3) is identical with that in equation (2) except that the exchange factor $F_{e-\mathrm{d}V_1}$ is replaced by $F_{e-\mathrm{d}V_2}$. However, these are essentially equal because the elements $\mathrm{d}V_1$ and $\mathrm{d}V_2$ are adjacent. This simply reflects the fact that the total flux passing through a unit volume is a continuous function as the interface between the regions is crossed.

Subtracting equation (3) from equation (2) gives the slip condition

$$(e_1 - e_2)_{\text{boundary}} = \frac{1}{4}\left[\left(\frac{Q_1}{\kappa_1}\right) - \left(\frac{Q_2}{\kappa_2}\right)\right]. \tag{4}$$

From examination of this equation, it appears that a slip in emissive power will always occur between the regions unless one of two conditions is met: either no sources or sinks occur in *either* region, or the ratio of source strength to the absorption coefficient is the same in each region. A special case of the latter condition is when both regions are identical, which, as expected, gives no slip.

Following the methods outlined in [1], the second-order diffusion solution can be used to derive the jump for the same case as equation (4). This approach gives

$$(e_1 - e_2)_{\text{boundary diffusion}} = \frac{3}{8}\left[\left(\frac{Q_1}{\kappa_1}\right) - \left(\frac{Q_2}{\kappa_2}\right)\right] \tag{5}$$

where the difference between equations (4) and (5) is obviously in the coefficient. It is interesting that a first-order diffusion solution predicts no discontinuity. Retaining the second-order terms allows inclusion of the effects of heat sources in the gas, and, hence, a slip is predicted. The larger slip for the diffusion case reflects the assumption inherent in diffusion solutions that the radiant flux is proportional to the gradient in emissive power even near boundaries.

For a gas that absorbs little radiation compared with that energy generated internally, the integrals on the right-hand side of equation (1) drop out. Fquation (4) is then obtained directly if the absorption coefficients for each region are replaced by their Planck mean values, given by

$$\kappa_p = \frac{\int_0^\infty \kappa_\lambda\, e_\lambda\,\mathrm{d}\lambda}{e}. \tag{6}$$

It does not appear possible, however, to derive a general slip condition for nongrey gases by using mean absorption coefficients except in these limiting cases. This is usually the case in spectrally affected problems.

For problems in which radiation is the dominant mode of energy transfer, the existence of this slip in gas emissive power at the interface between regions must not be overlooked.

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